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VALUE AT RISK AND EXPECTED SHORTFALL

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NICE-SOPHIA

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Introduction

Since the first day of finance, men wanted to know how much risk they were taking. How much they could lose over a day, or over a year. Everyone was making their indicator, their way of seeing, and explaining the numbers, of keeping money in case of a crisis.

We have to wait for the 90's to see a first risk standard indicator: the Value at Risk (VaR). In order to calculate it, most of the financial institution chose to base their calculation on historical data (on over a year). However, about 20 years later, the 2007 crisis showed us the limit of this indicator, because every financial product, every currency, is so correlated one with each other, that a crash is coming really fast, and that's an indicator based on the last year's data is not giving enough safety and enough information.

That is why financial institution moved to the stressed VaR, which is not looking for the last year's data, but the data from the worst year in the five last one. The new risk indicator was then composed by the regular VaR and the stressed one.

You can easily see the problem here: if the last year is the worst one, your risk indicator is based on twice the same data. This is where the Basel Committee gave a new recommendation that should be used in 2016: the Expected Shortfall (ES). The ES is an indicator that is giving both regular and stressed information. The point of this document is to explain the Value at Risk, the stressed VaR, and the Expected Shortfall and to explain how to implement an efficient ES calculation.

I. Value At Risk

1. Historic

In 1973, the Bretton Woods system was replaced by a regime based on freely floating fiat currencies. Following this changes, several crashes appears and the volatility explodes with the creation of derived product.

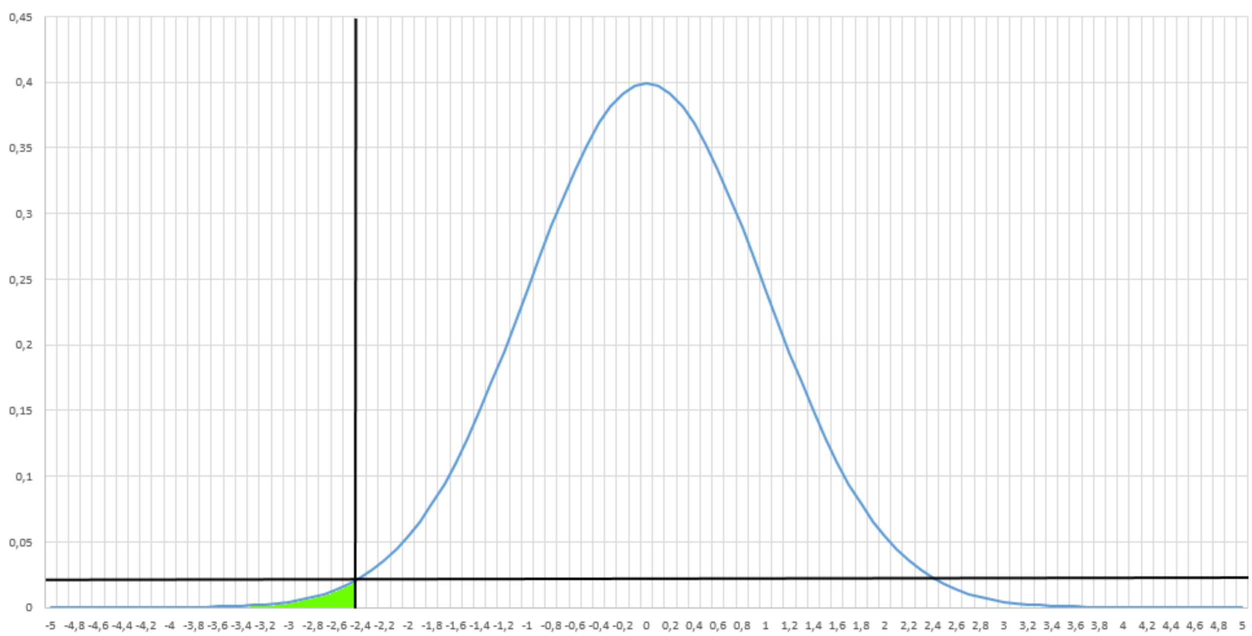
In the late 1980s, the Bankers Trust bank used for the first time the notion of Value at Risk. However, it is in the 90's, when JP Morgan created RiskMetrics, that the Value at Risk was really used. Sir Dennis Weatherstone, chairman at JP Morgan, wanted to have some daily reports measuring and explaining the firm's risks. Nevertheless, these indicators are specific to each market and not explicit.

We have to wait 1995 to see the Value at Risk really defined. This measure gives us a unique definition and a standard risk indicator and was part of the proposition of the Basel II Committee in 2004. It is one of many recommendation (like the utilization of the McDonough ratio instead of the Cook ratio) but made this indicator one of the most used in the financial institution.

2. Definition

Value at Risk is used to quantify the value of a portfolio's market risk.

It represents the potential maximal loss an investor may have on a given period with a given confidence level (probability). It gives you the worst loss amount you can expect over a defined period with a given a confidence level. For example VaR (95%, 1Day) is the value of the worst loss amount in one day with a 95% confidence level.



Graphic 1: VaR(95%,1)

3. Hypothesis

There are three main hypothesis:

1) The price of each asset follows a log-normal law. It is a necessary hypothesis and a strong condition.
2) A VaR with a period of N is equal to a VaR with a period of one time the square root of N.

$$VaR(X, N) = N \ \times VaR(X, 1)$$

3) The yield is null on the period of VaR.

   It is not a restrictive condition. Let an asset with a 15% yield. We consider change is open 262 days in a year. Daily yield is equal to 0.15*262=0.06%. This computation implies the daily yield is near to null.

4. Calculation

a. Historical VaR

First of all, we can compute VaR with an historical database. This method supposed that what was made in the pass will arrive again in the future.

It is very easy to use it. In fact, you sort your daily loss by value. Value at Risk given 95% on one day is the 95%th value. That means if you have only one hundred value, the VaR is the 95th value.

Most of the financial institution are using the Historical method in order to get the VaR.

Example

Let consider a period: 08/01/2014 to 08/01/2015. Our period is of one year when the ban is opening. We will look at Danone share price over this period. We compute P&L and then calculate VaR (99%,1). In order to compute VaR, we sort P&L with increasing order. We have the VaR by looking at the 99th value, ie in our case VAR (99%,1) =-1.41. This value means our loss will not exceed 1.41 with probability 99%.

08/01/2015	1.815
07/01/2015	1.74
06/01/2015	1.66
05/01/2015	1.52
02/01/2015	1.36
31/12/2014	1.35
30/12/2014	1.23
29/12/2014	1.22
23/12/2014	1.13
24/12/2014	1.13
...	...
16/01/2014	-1.17
15/01/2014	-1.29
14/01/2014	-1.38
13/01/2014	-1.41
10/01/2014	-1.49
09/01/2014	-1.57

Limit of this method

This method requires historical data and to consider an efficient period. The period should be not too short but not too large. A period of one year is considered as an efficient one.

Our example shows us one of the VaR's limits: it gives us information on how much time you are going to exceed a value, but it does not take into consideration if you just exceed it off a few amount or of a huge one.

b. Parametric Calculation

A model such as GARCH (Generalized Auto Regressive Conditional Heteroskedacity), RiskMetrics (1996) or Variance-Covariance method propose a specific parameterization for the behavior of prices.

A basic approach is a delta-gamma method. This method implies some hypothesis.

The first one is to consider that risk factors follow a normal distribution. Also it supposes that portfolio's risk is linearly dependant of risk factors.

Example

Let a portfolio estimate at one million with an annual volatility is equal to 20%.

It gives us a daily volatility equal to: $\frac{\sqrt{1}}{\sqrt{252}} \times \ 20\% = 1.2599\%$

We obtain :

$$VAR(99\%, 1D) = 1\ 000\ 000 \times 1.2599\% \times 2.33$$
$$= \mathbf{29\ 355}$$

Confidence Level	Centile of normal law
99%	2.33
97.5%	1.96
95%	1.64
…	…

This method is easy to use, but you need a covariance matrix. Unfortunately, covariance matrix should be re-compute at every second, this is not convenient. Moreover, assumptions are very strong, because we are not sure that portfolio's risk is linearly dependant of risk factors. A lot of derivate product do not respect this assumption, so we cannot use this method for many portfolios.

c. Monte-Carlo Simulation

The main idea of this method is to generate random data. These data represent likely P&L. This method should compute a very large number of simulations (at least 10 000). The number of simulations determine the precision of the quantile measure.

A very large risk factors are necessary in order to compute a good Monte Carlo's method. Therefore a complete portfolio revaluation is obviously too complex.

Example

Let's consider an asset, which realizations are going to be generated with Monte-Carlo simulation

Recall of the Black-Scholes formula: $dS = r\,S\,dt + \sigma S\,dW$

With :

- r : interest rate.
- σ: volatility
- W : brownian motion
- S : asset price

We obtain an explicit solution:

$$S(t) = S(0)\,e^{(r-\frac{\sigma^2}{2})t+\sigma W}$$

With an Euler discretization, we have:

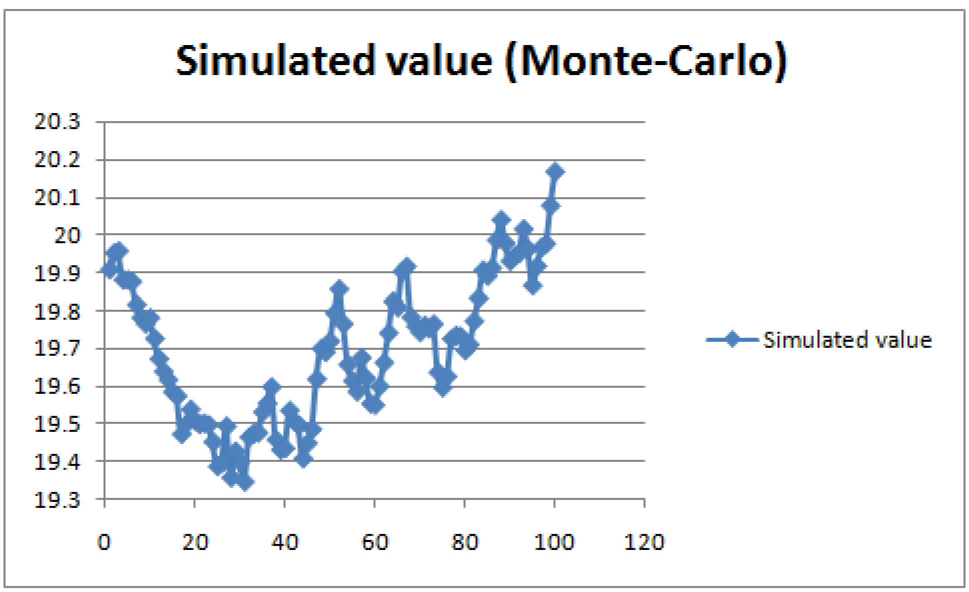
$$S(t+\Delta t) = S(t)\,e^{(r-\frac{\sigma^2}{2})\Delta t+\sigma W\sqrt{\Delta t}}$$

In order to generate a Brownian motion we use the Box-Muller method.

$$Y = \sqrt{log(U_1) * sin(2 * \Pi * U_2)} \qquad X = \sqrt{log(U_2) * sin(2 * \Pi * U_1)}$$

Where U_1 and U_2 are following a normal distribution.

We consider r=5%, σ=30%, dt=0.1. Then, we compute a hundred iterations and that give us the following realization:



Graphic 2: Monte-Carlo Simulated Values

The simulation give us data, we can use in order to compute VaR and ES.
Thus, we obtain a **VaR (99%) =-0.1363** and **ES (97.5%) =-0.1365**.
This method allows us to simulate various type of data.

	Pros	Cons
Historical	* Uses real data * Fast calculation	* Requires exploitable set of data
Parametric	* Fast to use * Quick set up	* Requires a model and to have the good parameter * Requires a covariance matrix * Strong Hypothesis
Monte-Carlo	* Lot of possible realization	* Really slow calculation * Requires a model and to have the good parameter

Comparison between the VaR calculation methods

d. Our Implementation's choice

We did not have to think a lot about this, because the choice to implement historic VaR was quite obvious for us. We previously saw that the parametric method is easy to compute, but that it is really difficult to have accurate and proficient parameters. The Monte-Carlo Simulation is a simulation. So even if you can say that the P&L are following a normal law, you still have to guess the parameters of that law and it is not based on real values.

That is why, and Auguste Claude-Nguestop from KPMG agreed with us to choose the historical estimation. In our point of view, Historic estimation is more adapted to VaR's computation. Indeed, instead of supposing that P&L follow a normal law and guessing the parameter, you use real data. One of the cons for using this method was the need of various data in order to test our algorithm. But KPMG gave us P&L for various equities and change rate for over ten years in order to let us work, test and back-test so it was not a real cons in our case.

5. Limits of the VaR

The VaR had some limitation that could be catastrophic in some case, because they were no information about the amount of loss that was exceeding the VaR, so it did not take into account the extreme loss that we could still have.

Another main problem is that the VaR is mostly considered the last year data, so it can "forget" how quickly and how a disaster a crisis such as the subprime is.

In a Mathematical point of view, the VaR is not easy to use because it does not respect the subadditivity axiom (if you add the VaR of a portfolio A and of a portfolio B, you are not always higher than the VaR of a portfolios “A+B”).

This is clearly a problem if you are looking at a really diversified patrimony, because we should have $VaR(A+B) \leq VaR(a) + VaR(b)$ and the VaR does not respect this.

II. Stressed Value at Risk

1. Definition and formula

In order to take into account a crisis period, we calculate the stressed VaR (SVaR) based on what happened during a complicated year.

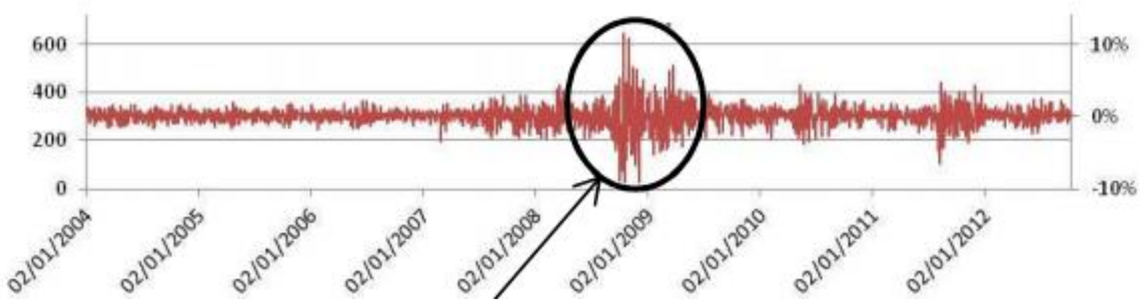
This risk metric was required to avoid huge losses by having not anticipate a crisis. In today’s world, the crisis is more and more unpredictable and a risk metric that is taking into account previous one (or simulating a crisis) is a must have and must be taken into consideration.

2. Calculation

The SVaR can be calculated using the same method we previously used for the usual VaR.

a. Historical Method

To calculate the SVaR by an historical way, we take the value of a year that is representative of a crisis year (depending on the asset we are looking at, it can be for example the 2007 subprime crisis) and then use the same method used for the VaR calculation using an historic method.



Graphic 3: Stressed Period we would take for the given asset

b. Parametric & Monte Carlo

In order to simulate a crisis, you can use the same formulae as for the regular VaR with adjusted parameter/coefficient (bigger volatility for example) in order to fit to a crisis time.

3. Stressed VaR utilization

The SVaR is a complement to the VaR. Most of the time, the metric used is an aggregation in some defined proportion of the VaR and of the SVaR. It allows you to have a metric that take into account the previous days (with the regular VaR) and also a part in case we have a crisis (with the SVaR).

4. Limits of the Stressed VaR

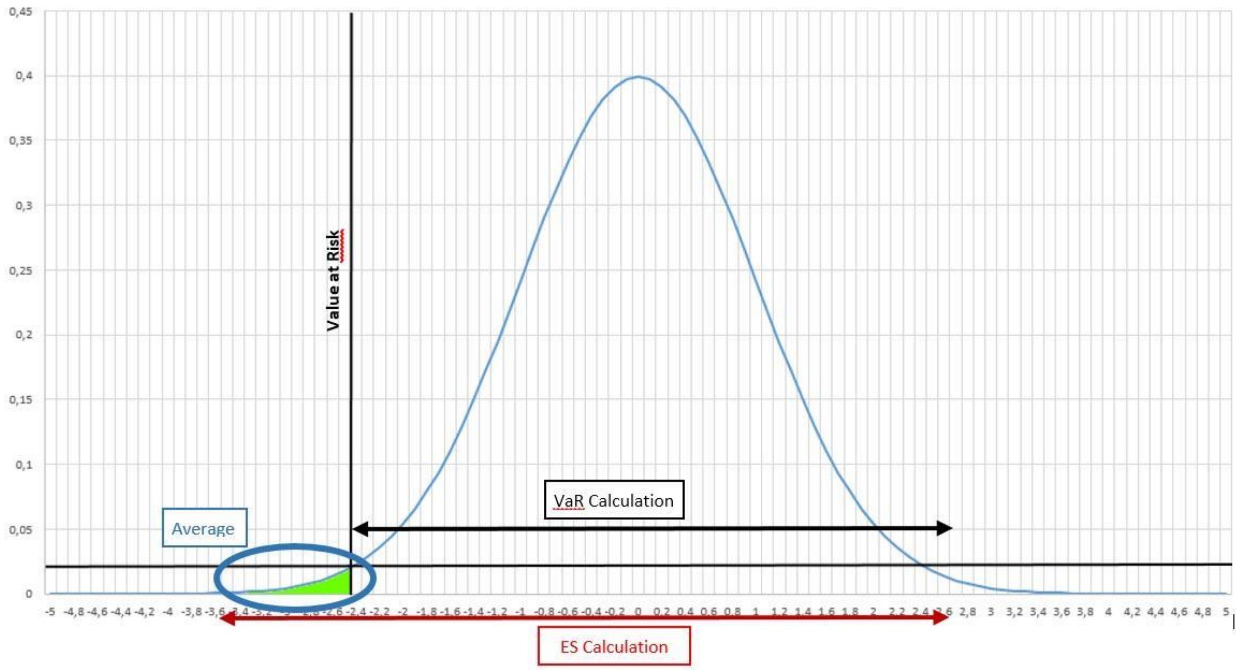
The Stressed VaR eliminate a bad points of the VaR by enabling the indicator to take into account a crisis. However, the same calculation is made so it is still a non-subadditive measure and do not take into account extreme losses.

Also, it required, in case that you use the historical method, to have a very large set of data in order to have a crisis to base your calculation on.

III. Expected Shorfall

1. Definition and formula

The ES is the average value of the loss that are exceeding the VaR.



Graphic 4: Difference between VaR and ES

So the value of the Expected Shortfall with a probability α is:

$$ES_\alpha = E(X \mid X \leq VaR_\alpha(X))$$

The point of this metric is to have a metric close to the VaR but that take into account the maximal loss (loss that are worse than the VaR). That is also allowing you, if you take a longer period to have a history over the crisis and to have some indication on some stressed periods.

2. Calculation

In order to calculate the ES, we can use exactly the same methods as for the regular VaR: Historical, parameterization or Monte-Carlo.

Here I will just use the example of the calculation of the VaR we previously use to explain the calculation of the VaR with the Historical method.

Example for historical method

Previously, we found out that the VaR (99%, 1Day) between the 08/01/2014 and the 08/01/2015 was 1.41.

So in order to calculate the ES, we just have to look at the average of the value exceeding 1.41:

03/03/2014	-1.29
05/01/2015	-1.38
24/01/2014	-1.41
25/09/2014	-1.49
12/12/2014	-1.57

Which give us:

$$ES_{99\%} = (1.41 + 1.49 + 1.57)/3 = \mathbf{1.49}$$

3. Subadditivity demo

VaR is not subadditive. That means you are not sure to reduce your risks by diversifying your portfolio. In other words, diversification don't reduce risk even if this is obviously counterintuitive. The risk measure Expected Shortfall is subadditive. It allows to approximate very easily an aggregate portfolio because:

$$ES(A+B) \leq ES(A) + ES(B)$$

Obviously, we could compute *ES (A)* et *ES(B)* with a confidence level of 97.5%.

Let $L_1$and L_2losses of two different portfolios. We assume the fact that $E(L_i) < \infty \; and \;\; i = 1{,}2$.

We want to show that: $ES(L_1 + L_2) \leq ES(L_1) + ES(L_2)$

That means $E(L_1 + L_2 \mid L_1 + L_2 \geq VaR_{L_1+L_2}\ ) \leq ES(L_1 \mid L_1 \geq VaR_{L_1}) + ES(L_2 \mid L_2 \geq VaR_{L_2})$

$$E(L_1 1_{L_1 \geq VaR_{L_1}} + L_2\, 1_{L_2 \geq VaR_{L_2}} - (L_1 + L_2)\, 1_{L_1+L_2 \geq VaR_{L_1+L_2}}\ ) = \sum_{i=1}^{2} E(L_i(1_{L_i \geq VaR_{L_i}} - 1_{L_1+L_2 \geq VaR_{L_1+L_2}}))$$

If we add and we subtract $VaR_{L_i}(1_{L_i \geq VaR_{L_i}} - 1_{L_1+L_2 \geq VaR_{L_1+L_2}})$

That give us :

$$\sum_{i=1}^{2} E(L_i(1_{L_i \geq VaR_{L_i}} - 1_{L_1+L_2 \geq VaR_{L_1+L_2}}))$$
$$=$$
$$\sum_{i=1}^{2} (E((L_i - VaR_{Li})(1_{L_i \geq VaR_{L_i}} - 1_{L_1+L_2 \geq VaR_{L_1+L_2}})) - VaR_{L_i} E(1_{L_i \geq VaR_{L_i}} - 1_{L_1+L_2 \geq VaR_{L_1+L_2}}))$$

We have: $E(1_{L_i \geq VaR_{L_i}} - 1_{L_1+L_2 \geq VaR_{L_1+L_2}}) = 0$, due to the fact that we compute the VaR with the same confidence level.

And $(E((L_i - VaR_{L_i})(1_{L_i \geq VaR_{L_i}} - 1_{L_1+L_2 \geq VaR_{L_1+L_2}})) \geq 0$because it is an expectation of a product. Each member of the product has always the same sign.

Thus, we obtain expected shortfall is subadditive.

4. Basel III Committee

The 2007 subprime crisis highlighted the limits and the weaknesses remaining in the regulation measure taken by the Basel II Committee. It was an obligation to fill those gaps and to regulate financial markets and at the same time give some recommendation about risk measurement.

Published in December 2010, the main principles are to strengthen financial institutions in order to ensure the bank liquidity and to decrease the bank leverage while changing the risks indicators in order to better fit to the financial markets.

It is in that optic that the Basel III Committee agreed to replace the Value At Risk with the Expected Shortfall for the internal model-based approach.

They also had to recalibrate the level of confidence in order to stay consistent. That is why, instead of using the 99% level of confidence like for the VaR, the Basel III Committee recommends to use the 97.5% level of confidence for the ES. It allows you to capture the same risk level.

5. Stressed ES

As for each risk measure, there is also a stressed calibration of the Expected Shortfall risk measure. For internal model, Basel III Committee propose to compute an Expected Shortfall on a duration of ten years. But even over a long duration, we are not ensuring that every risk factors are stressed in the observed period. It is also not efficient to look on a period where the full set of risk factors is stressed. In order to avoid that issue, The Basel Committee suggests to compute a combined method. The main idea is that bank specify a set of relevant risk factor. This set depends on the risk exposure of each portfolio. In order to have a relevant set, the bank should have enough long observations, the set is not relevant if it requires to compute some approximation to fill the data. The freedom of the bank to choose a set of risk factor is limited. In fact, the set of risk factor should be explained at least 75% of the ES model.

The process advised by the Basel Committee is to compute the Expected Shortfall based on the aggregate bank's portfolio. The formula is the following :

$$ES = ES_{R,S} * \frac{ES_{F,C}}{ES_{R,C}}$$

The first expression $ES_{R,S}$ is the Expected shortfall using the reduced set of factor. The period should be the most stressed periods, it depends on the set of risk factor and it is relative to the bank's portfolio. The second $ES_{F,C}$ represents the Expected Shortfall using the full set of factor in the current period. The duration of the Expected Shortfall shouldn't be very large, a 12th month period is fine. The last expression of the computation $ES_{R,C}$ means the Expected Shortfall using a reduced factor's set on the current period. Same as the second expression, the Basel Committee advises to use the most recent 12th month period. The Expected Shortfall will be estimate at a 97.5 percentile of the distribution, it ensures to have an efficient risk measure.

6. Limits of the ES

Unfortunately, the Expected Shortfall has various limits.

The Expected Shortfall considers the entire distribution of assets. The last centile is necessary in order to calculate the ES where it was not required to calculate the VaR.

The biggest limit is that data required must be over a long period in order to compute ES. In case you compute ES (1D, 99%) on a duration of one year, you only have three value to compute your average. In other words, your sampling is too short to compute a representative ES. The Basel III committee advises to have a ten year period to calculate the ES. This constraint could be strong, because you need a very large sampling. If you use a Monte-Carlo simulation, a ten year prediction is too long and would require too many calculation in order to simulate some value over that period. Thus, it is strongly advising to have historic data. Most of Bank or big company can have access to these data, nevertheless all financial actor do not have access to it. The second consequence to a large duration is that it is very difficult to use back testing. Indeed, you need a duration of twenty year to make a useful back testing! It is clearly not possible. The second possibility would be to use back testing over smaller duration, for example computing an ES on eight year and to use a back testing over two year. Unfortunately, as you will see in the Back testing part, it is not efficient.

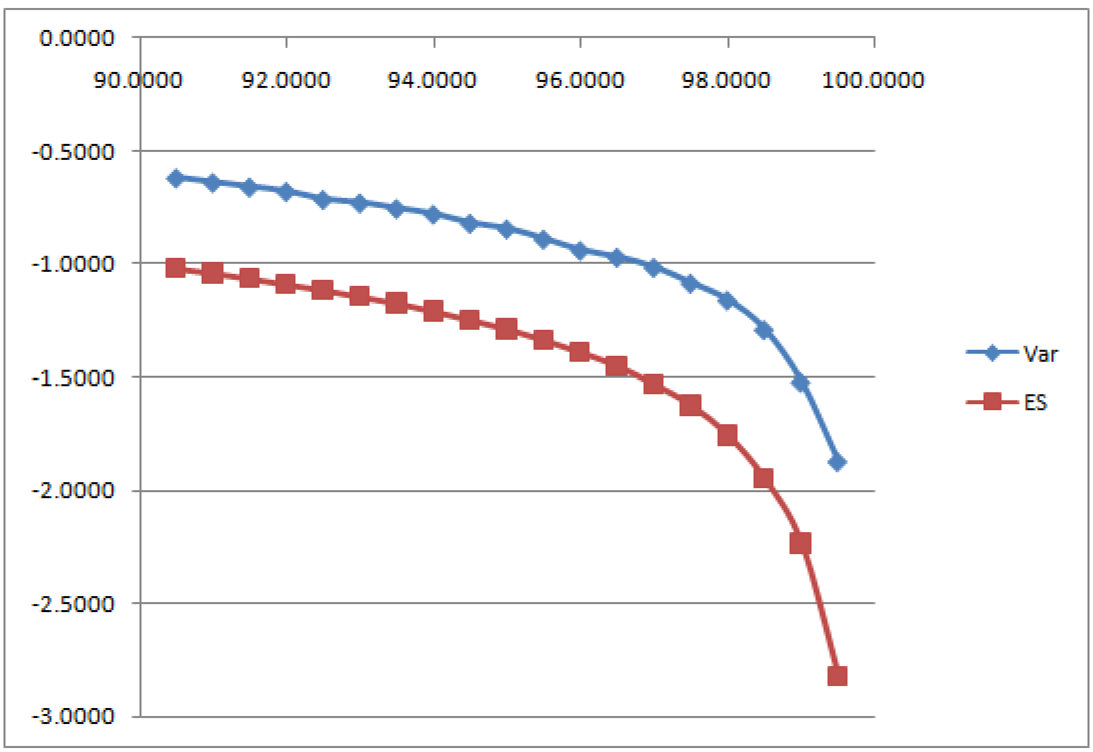
IV. Analysis

In this part, we will analyze the relation between VaR and ES.

Firstly, we focus our analysis on equities with a confidence level between 90% and 99.5%. Confidence level increase by step of 0.5% and we analyze the evolution of the VaR and of the ES.

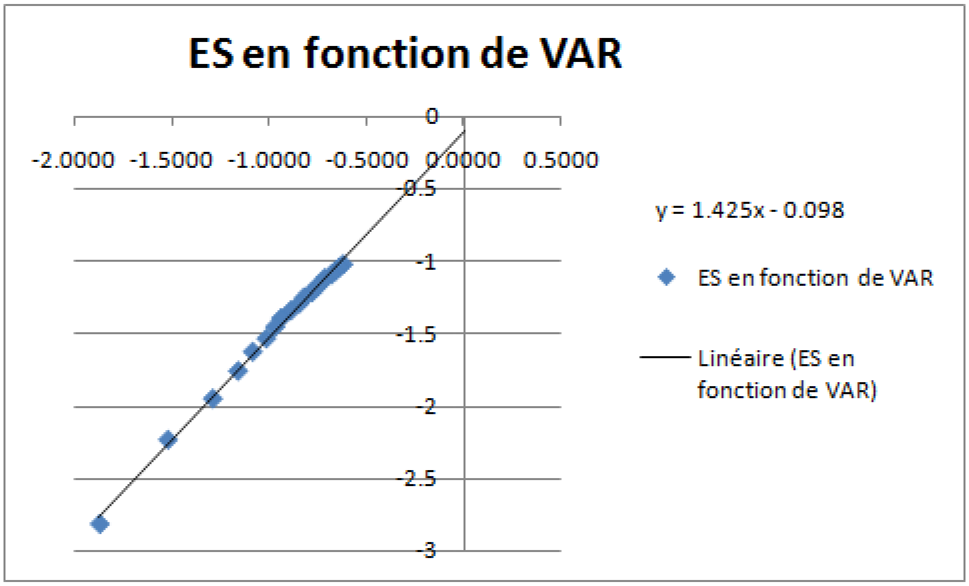
In order to have a complete analysis, we have done our analysis on several equities (Danone, Barclays, Airbus…). All of them are big company but they are diversified. During our analysis, we note that VaR and ES behavior are the same for each equities. Thus, we could generalize our analysis to all equities.

Let take the Airbus example. We compute an historic VaR and ES on a duration of nine years (12/2005-12/2014) for different confidence level. Then we obtain the following graph :



Graphic 5: VaR and ES by confidence level

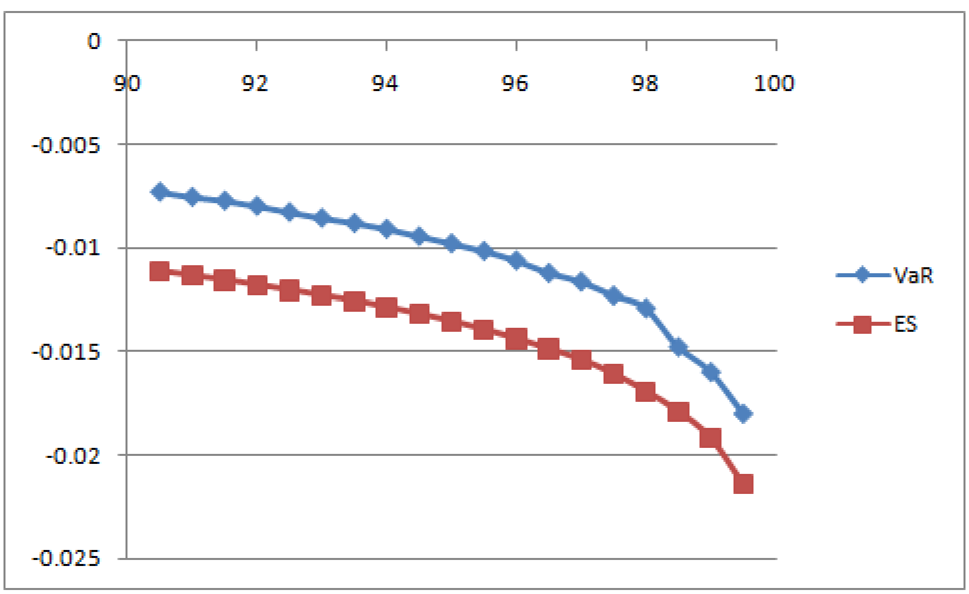
We can see that the behavior of the VaR and of the ES are the same. They have the same variations. In order to confirm our hypothesis we illustrate dependence between the ES and the VaR.



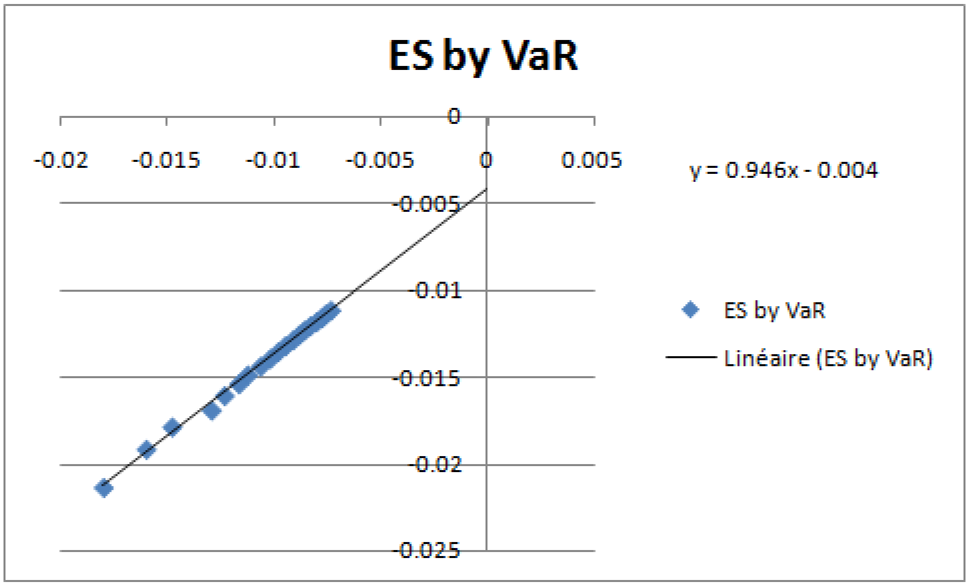
Graphic 6: ES by VaR

By a linear regression, we show that we can get ES from the VaR. That means that the ES follow same variation of VaR. Thus, we have a risk measure that is closer to the real loss but still following the VaR behavior.

We compare our first analysis with a second analysis on rates exchange. We study the EUR/USD exchange rates on a duration of nine years (12/2005-12/2014).



Graphic 7: VaR and ES by confidence level



Graphic 8: ES by VaR

We can have the same conclusion we made on the equities, the behavior of ES and VaR is very similar.

The next step of our analysis is to confirm the following hypothesis given by Bale III: The quantity of risk measured by a 99% VaR is approximately the same as a 97.5% ES.

We compute an historic VaR and ES for each confidence level between 90.5% and 99.5%.
To do so, we use the Barclays auction price from 12/2005 to 12/2014.

Confidence Level	VaR	ES
90.5	-9.67	-16.1051
91	-10.162	-16.4351
91.5	-10.346	-16.8134
92	-10.795	-17.1877
92.5	-11.245	-17.5807
93	-11.695	-18.0352
93.5	-12.145	-18.4791
94	-12.701	-19.0051

94.5	-13.255	-19.536
95	-13.944	-20.1638
95.5	-14.456	-20.7938
96	-15.293	-21.5767
96.5	-15.969	-22.3911
97	-17.089	-23.4173
97.5	-18.428	**-24.5446**
98	-19.342	-26.0331
98.5	-21.246	-27.9719
99	**-24.248**	-30.633
99.5	-29.238	-34.7416

In this case the equivalent of a Var(99%) is a ES(97.5%), we check with other equities and we have similar result.
Thus, we are able to confirm that this assumption is true.

V. Excel Template

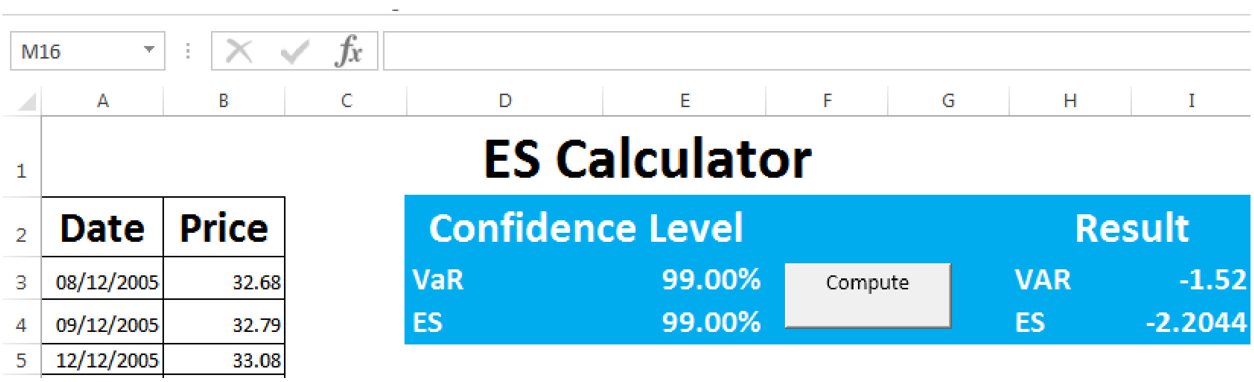
Available at https://drive.google.com/file/d/0B55M8s30A9phcW42dWM0c2lfTFU

1. Specification

In order to make our tests and to first well understand every concept, but also to allow us making quick analysis, we began with an excel file that was calculating the ES and VaR with given data.

That allow anyone to have a quick access from anywhere to a ES and VaR calculator.

2. Production



Screenshot of the Excel Template

We manage to develop this tool that is supposed to be really simple to use. Just copy/paste your data in the Date/Price column and choose your confidence level. Then just click on compute to get your results.

VI. Web Application

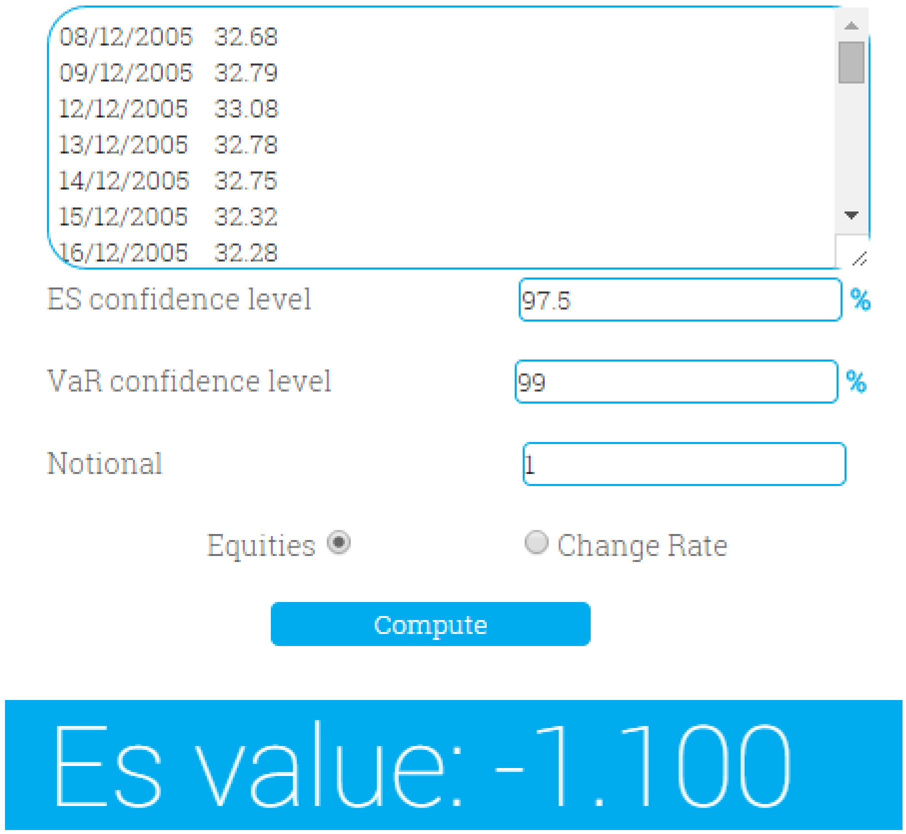
Available at http://users.polytech.unice.fr/~dufour/ESCalculator/ .

1. Specification

Auguste Claude-Nguestop from KPMG asked us to give him a web application that would allow him and his clients to compute quickly the expected shortfall for a given notional without any technical specification.

In order to have a tool that would be quickly deployed and that could be used even off-line, we choose to use only HTML/CSS/JS, without any server side.

2. Production



Screenshot of our Web Application

Our tool was designed to be as simple as possible. We just ask the ES and VaR confidence level (to know what it is, please look the description of ES and VaR in the previous part of our document) and the notional. Then, once you have given the data in the right format, you just have to click on compute to have the ES and VaR value.

In order to show how to use our tool, we let the user access to an example he can have by just clicking on example. This will fill the form with our test data.

Our page is a static one, even if we are using a form, there is just some javascript behind and all the calculation is made in the javascript program.

VII. Back-Testing

In order to check that the ES is useful and is a real risk indicator, we are going to make some several tests. We have the data for Airbus/Barclays/Danone equity and the EUR/USD for over 10 years, so to check whether or not our calculation is meaningful, we will calculate the ES on the first eight years and then check regarding to the ES on the last two years.

Then we calculate the error between what we would have expected (ES/VaR over the eight first years) and compare it with the effective ES/VaR during the last two years.

ES (97.5%) and VaR (99%):

Nom	VaR (2005-2013)	ES (2005-2013)	VaR (2013-2015)	ES (2013-2015)	VaR Error	ES error
Airbus	-1.230	-1.405	-1.969	-2.161	0.739	0.756
Barclays	-25.189	-26.185	-12.47	-12.5	12.719	13.685
Danone	-1.9939	-2.075	-1.490	-1.522	0.449	0.553
EUR/USD	-1.6757%	-1.6927%	-1.1305%	-1.1439%	0.5318%	0.5622%

ES (95%) and VaR (99%):

Nom	VaR (2005-2013)	ES (2005-2013)	VaR (2013-2015)	ES (2013-2015)	VaR Error	ES error
Airbus	-1.230	-1.125	-1.969	-1.7	0.739	0.575
Barclays	-25.189	-21.705	-12.47	-10.019	12.719	11.686
Danone	-1.9939	-1.667	-1.490	-1.273	0.449	0.394
EUR/USD	-1.6757%	-1.4287%	-1.1305%	-0.9844%	0.5318%	0.4443%

First of all, let's check our result. We can obviously see that the ES (97.5%) is bigger than VaR (99%). Moreover we check our method with manual computation to be sure that the value our tools gave us were right, which was always ok.

Then, we have done a back testing of ES computation on various duration. We compute ES and VaR over an eight years duration, and then we compare and compute errors with another calculation over two other years. That give us the previous tables. In case of an ES with a confidence level of 97.5%, the VaR (99%) is nearest than the ES. Nevertheless, ES with a confidence level of 95% has a smaller error. It is not possible to generalize for all equities. The 2008 crisis doesn't affect each equities at the same time and at the same level. Moreover, a VaR with a duration of two years is not an abnormality but an Expected Shortfall over two year is totally inefficient. It is the main problem of the Expected Shortfall, it is really difficult to do a back testing. That means you are not able to check if your risk measure is coherent with real market.

How can we explain this fact? A very important thing about backtesting, is that for a measure to have a meaningful backtesting, it should be elicitable.

A definition from Dirk Tasche in his presentation "ES is not elicitable – so what?" is:

"The functional is elicitable relative to P if and only if there is a scoring function s which is strictly consistent for relative to P."

A possible interpretation is that points from an elicitable functions can be determined with a regression. And this is where the link between elicitability and backtesting shows up: how could a measure gave some results for a first period that would "predict" the ones over a future period if there was no regression possible?

If ν is elicitable, then we have:

$$\forall \pi \in [0,1], \qquad t \in \nu(P_1) \cap \nu(P_2) \Rightarrow t \in \nu(\pi P_1 + (1-\pi)P_2)$$

And that is not true for the Expected Shortfall, so it is not an elicitable measure, and so will not give a meaningful backtesting.

Conclusion

The Value at Risk is a simple risk measure. It allows us to have a first idea of the value we can lost. However, during the 2008 crisis, VaR show us limits. The committee Basel III encourages to use the Expected shortfall. First of all, the Expected Shortfall is a coherent risk measure, it respects a lot of mathematics rules such as subadditivity… It is easy to rebuild his portfolio and compute the Expected Shortfall. Then by nature Expected Shortfall consider the entire distribution of the equity value. By his properties the Expected Shortfall is more representative of your possible loss than the Value at Risk. In some case, such as your lasts centiles is very scattered, the Expected Shortfall is necessary and the Value at Risk is very far of real risk. Nevertheless, as we have shown in the backtesting part, the Expected Shortfall is not a miracle solution, due to a crisis the value of the Expected Shortfall is still far from your real possible loss.

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